Benefits of Non-Linear Stress-Strain Properties & Membranes
For RCC Dam Stresses

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ABSTRACT
Using the non-linear stress-strain properties, instead of assuming an incorrect but more traditional constant linear modulus of elasticity, provides a true picture of actual stresses in the analysis of dams constructed with low cementitious content RCC. This allows strain softening and load redistribution, with reduced peak stresses and more uniform stress distribution at the foundation.

An impervious upstream membrane and face drain system with its ability to control uplift pressures at the upstream face and heal of the dam can also be included in state-of-the-art stress analyses. This further reduces peak stresses and improves stability.

INTRODUCTION
The typical stress-strain behavior of traditional concrete and high cementitious content RCC results in a reasonably constant modulus of elasticity that is valid for stresses ranging from zero to near ultimate strength. It is therefore acceptable, with this type of concrete, to simply use a single value of modulus in stress calculations for all locations and levels of stress in the dam.

However, lean RCC mixes typically have non-linear stress strain behavior with substantial softening after stresses exceed about 25% of the ultimate strength. This allows redistribution of loads from high stress areas such as the faces and heel of a dam to lower areas of stress towards the interior of the dam. Peak stresses are therefore decreased.

The reduction of modulus with increasing stress can be used as input to a non-linear stress program, or a linear program can be used by iterating an FEM analysis which replaces the initial or previous modulus values with the correct modulus for the stress levels indicated in each iteration. When this was done for Mujib dam, approximately four iterations were required, with a substantial reduction of stresses being realized.

The Mujib dam analysis also investigated the structural benefits of an upstream membrane and face drain system to control seepage and uplift. The membrane provided an additional substantial reduction of peak stresses. The analyses included no membrane (0% efficiency), a membrane that is 50% efficient, and a membrane that is 100% efficient.

Mujib dam is a 60 meter high RCC dam constructed with a relatively lean RCC mix having a cement content of 85 kg/^{3} and no pozzolan. The upstream slope is 0.1:1.0, and the downstream slope is 0.8:1.0. A gallery is located about 5 meters above the foundation.

NON-LINEAR STRESS-STRAIN BEHAVIOR
A separate companion article, *Mix Designs and Properties of RCC at Mujib Dam – High and Low Cementitious Contents*, discusses basic material properties of the RCC used in Mujib Dam. It includes a chart showing the elastic and secant modulus of the RCC as a function of stress level and strength. The article also contains a table showing the modulus as a function of age.   Significant strain softening occurred.   This is typical of low cementitious content RCC. Figure 1 shows a typical stress-strain curve at 1 year for RCC placed in Mujib dam.

Figure 1
Stress-Strain and Secant Moduli at 1 Year for RCC Placed in Mujib Dam
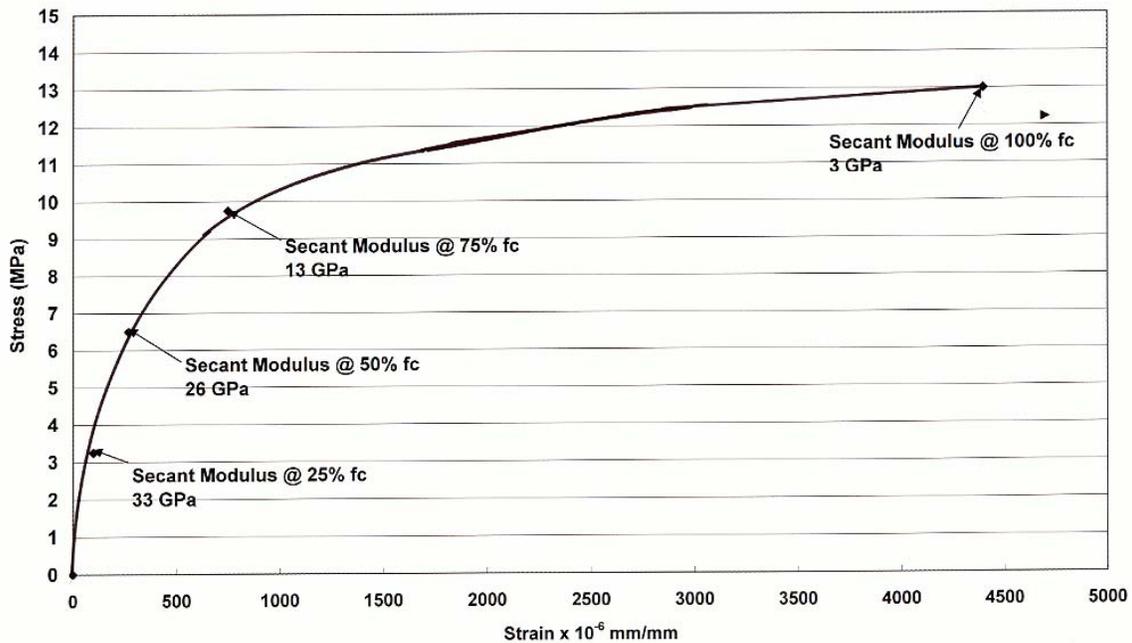
The dramatic beneficial strain softening attainable with lower cementitious content RCC is also obvious in Figure 2. It compares the stress-strain behavior of higher and lower cementitious content RCC at Big Haynes Creek dam in the USA. Although strain softening is primarily related to the decrease in cementitious content, it can also be influenced by the quality of aggregate, moisture content of the mix, aggregate gradation, quantity of aggregate fines, and characteristics of the aggregate fines.

Figure 2
Example of Stress-Strain and Modulus for Higher and Lower Cement Content RCC
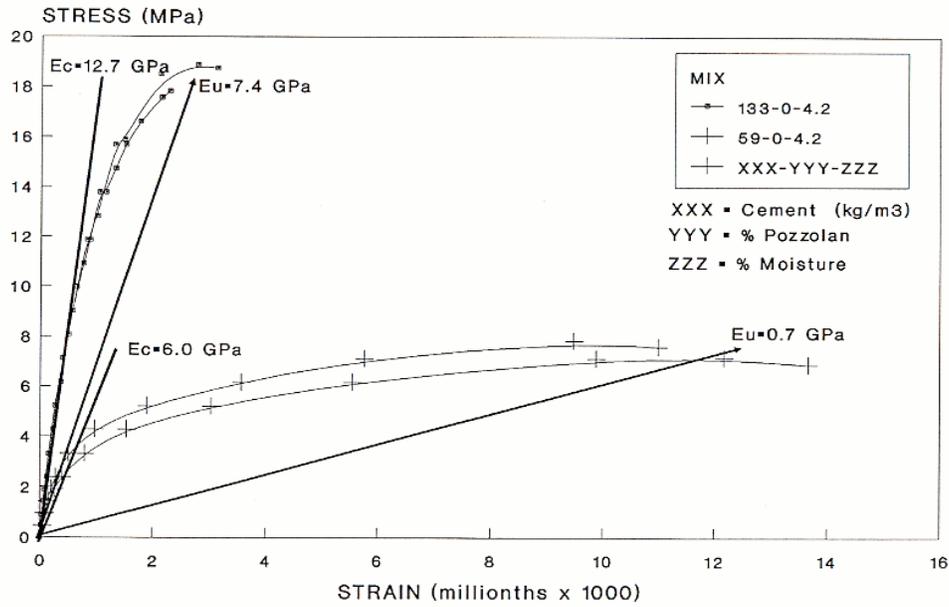

Figure 3 compares the stress-strain behavior and modulus of RCC at different ages and conventional facing concrete at Mujib. The facing mix has an essentially linear stress-strain curve and high modulus typical of traditional concrete and high cementitious content RCC, while the low cementitious content RCC again has low modulus and strain softening.

Figure 3
Stress-Strain of RCC at Two Ages and Conventional Concrete at Mujib Dam
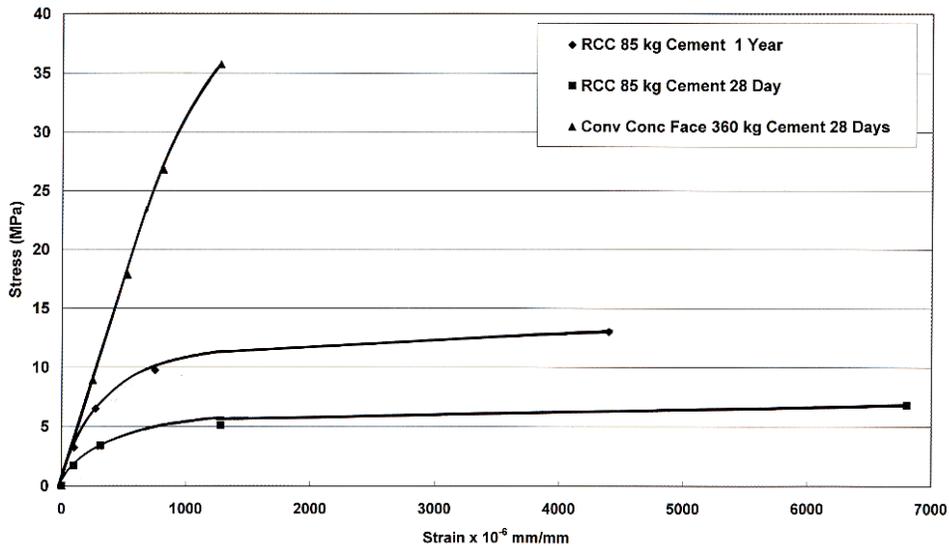
COMPRESSIVE vs. TENSILE STRESS-STRAIN BEHAVIOUR

Obtaining stress-strain curves is relatively easy with compressive test specimens. However, it is difficult to get reliable stress-strain curves under tension. Fortunately, the non-linear stress-strain behavior under compressive load is similar to what occurs under tensile load. This has been established in detailed testing at other projects. Figure 4 shows a comparison of test results for cores of mass samples made with one of various cementitious contents for Balambano dam in Indonesia.

Figure 4
Comparison of Stress-Strain Behavior in Tension and Compression
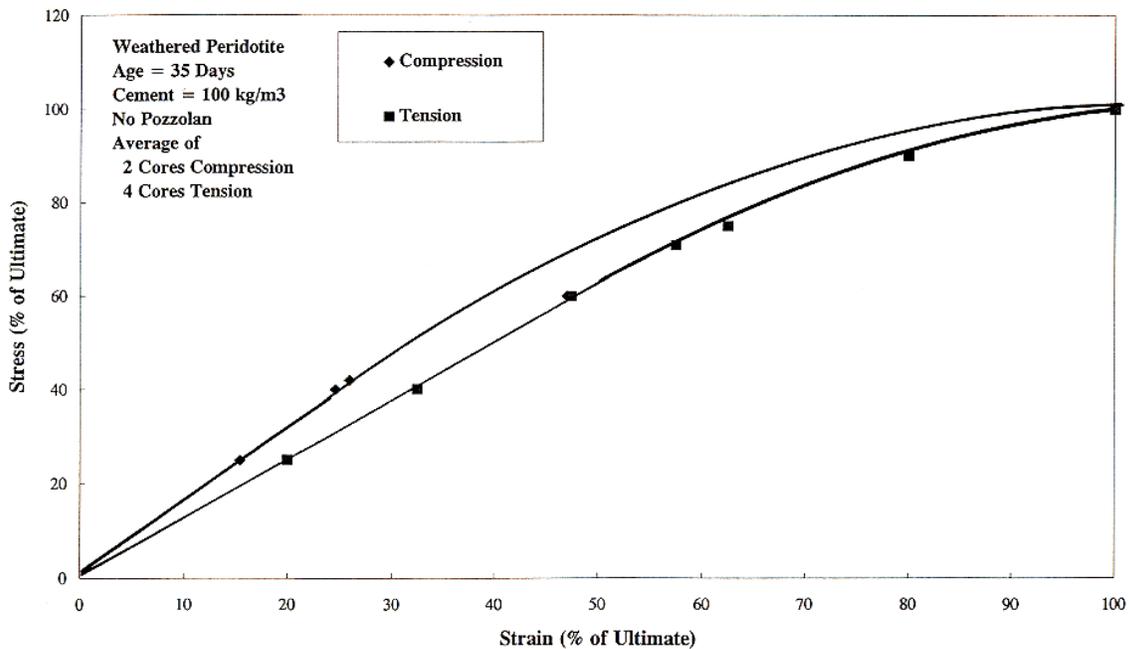
STATIC & DYNAMIC MODULUS VALUES FOR MUJIB DAM

Table 1 compares the estimated modulus values used for the stress analysis of Mujib dam to the average of test results for RCC actually placed in the dam. Some early mix studies with blend sand indicated different modulus values, but further evaluation and testing resulted in the more credible values indicated in Table 1.

The dynamic modulus under earthquake loading is a higher value than the static modulus. Under very fast load or strain rates the material acts as if it were stiffer. For strain rates typically experienced in a dam under earthquake loading, the dynamic increase factor (DIF) for modulus can be approximated at 180% for concrete having 0 to 7 MPa, 140% for concrete having 7 to 14 MPa, and 130% for concrete having more than 14 MPa. Because the ultimate long term strength of RCC in Mujib dam expected to be on the order of 14 MPa, a factor of 135% of the static modulus was used for the dynamic modulus in the stress analyses.

Table 1
Estimated Modulus Values For the Stress Analysis vs. Actual Tests Results

Stress Level (% of Ultimate Strength)	Modulus (GPa) at 1+ Year		
	Results From Tests of RCC in the Dam Static	Estimated Values Used In Stress Analysis	
		Static	Dynamic
0%	33		
0 % to 25 %	33	33	45
25 %	33		
25 % to 50 %	29	26	35
50 %	24		
50 % to 75 %	18	19	25
75 %	13		
75 % to 100 %	8	11	15
100 %	3		

## NON-LINEAR ANALYSIS USING LINEAR PROGRAMS

The easiest way to include the non-linear stress-strain behavior of RCC in a stress analysis is to use a program that allows the non-linear modulus function to be defined. However, FEM programs that can only accept a single modulus value can also be used. This was done at Mujib with the program SAP90. After starting with a single elastic modulus value assigned to all locations in the FEM mesh, adjustments were made to subsequent computer runs using the modulus value appropriate for the level of stress indicated by the previous stress output for each location in the dam. At Mujib, stresses stabilized after about 4 iterations. No changes to the FEM mesh or other input were required.

Because all stresses for static load conditions at various reservoir levels without earthquake resulted in stresses less than 25% of the static ultimate tensile and compressive strengths, only one modulus value at 33 GPa, appropriate for 0% to 25% of ultimate strength, was actually used for these load conditions. However, adjustments were made to the modulus values for various elements for the dynamic load condition where tensile stresses were high. Table 1 shows the dynamic modulus values for various levels of load, and Figure 5 shows the distribution of those modulus values in the final FEM iteration.

Similar to the dynamic increase factor for modulus, RCC undergoes a dynamic tensile strength increase factor. A factor of 150% was used at Mujib. This is an industry standard for traditional dam concrete. In reality, the dynamic increase factor is greater for lower strength concrete, and greater for higher strain rates. Using the 150% factor was, therefore, a conservative approach at Mujib. The benefits of lower cementitious content RCC with its increased tensile DIF is the subject of a separate article.

Stresses for static load conditions should be compared to static strength, currently expected to exceed 1.4 MPa tensile in the unjointed RCC mass and lift joints with bedding at Mujib. Stresses for dynamic load conditions should be compared to dynamic strengths, currently expected to exceed 2.1 MPa [150% x 1.4 MPa] in the unjointed RCC mass and lift joints with bedding. Lift joints without bedding are currently expected to have a static tensile strength of about 0.5 MPa, with a dynamic tensile strength of about 0.7 MPa.

Figure 5
Modulus Values Used in Mujib Dam Stress Analysis

Static Modulus 33 GPa Entire Dam
Dynamic Modulus (GPa) As Indicated
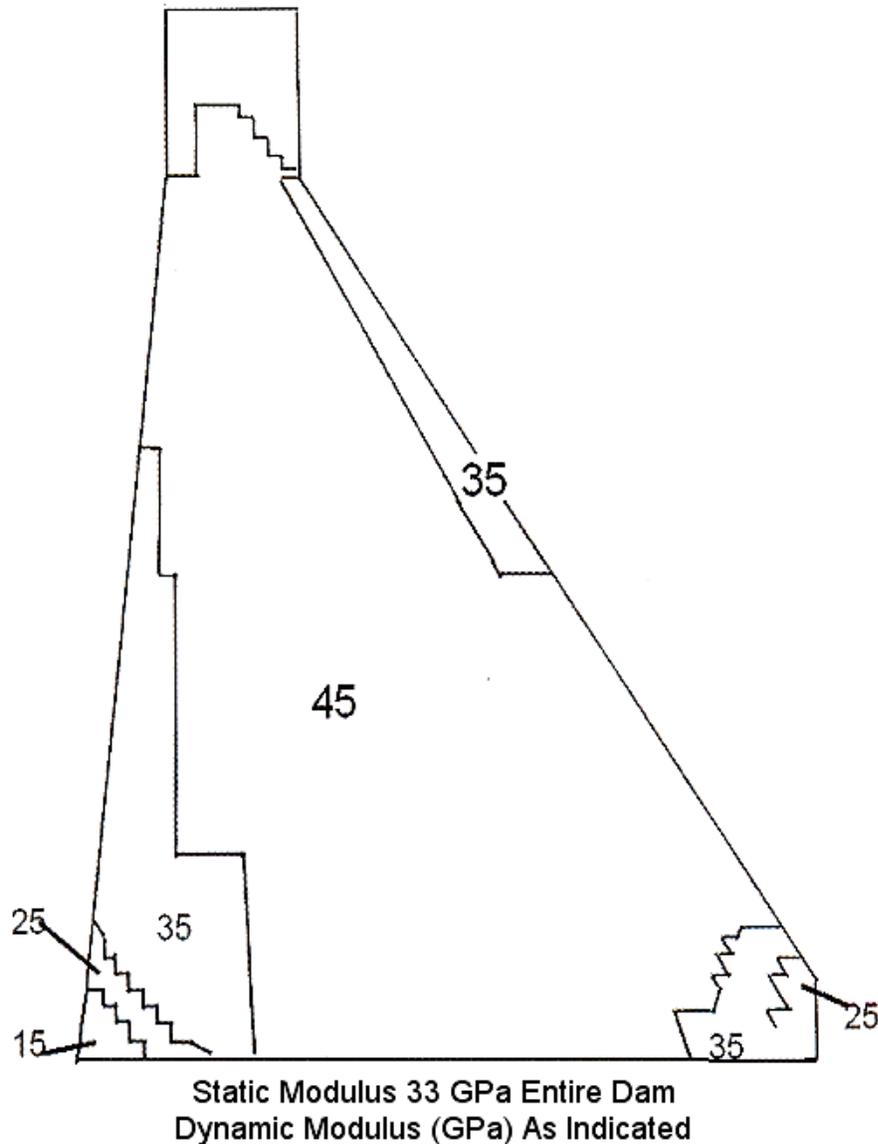
MUJIB DAM STRESSES WITH & WITHOUT STRAIN SOFTENING
There is insufficient space to show results of the many static, OBE, and MCE load conditions that were analyzed at Mujib, including various efficiencies for the upstream membrane. Vertical tensile stresses across lift joints are usually the controlling factor in RCC dams, so Figures 6 and 7 were selected to compare maximum vertical stresses (tension) for the first and last stress iterations under the MCE load with no membrane. Static load stresses including uplift + earthquake load stresses are indicated. An uplift reduction of 67% was taken at the line of drilled internal drains. Figure 6 shows stresses for a constant modulus of 33 GPa and a constant dynamic modulus of 45 GPa (no strain softening). Figure 7 shows stresses when strain softening under the dynamic load condition was considered, using the modulus values indicated in Figure 5.

The figures show that loads are re-distributed from the face and heel of the dam to the interior. For example, peak stresses that were about 1.9 MPa in the heel reduced to about 1.4 MPa as the modulus was adjusted, and a large region of the upstream face that had tension on the order of 0.5 to 0.7 MPa reduced to about 0.3 to 0.5 MPa

Figure 6
**Vertical Stresses without Strain Softening - No Upstream Membrane**

Vertical Stress
Static + 100% Uplift + Dynamic (MCE)
Modulus = 33 GPa Static, 45 GPa Dynamic
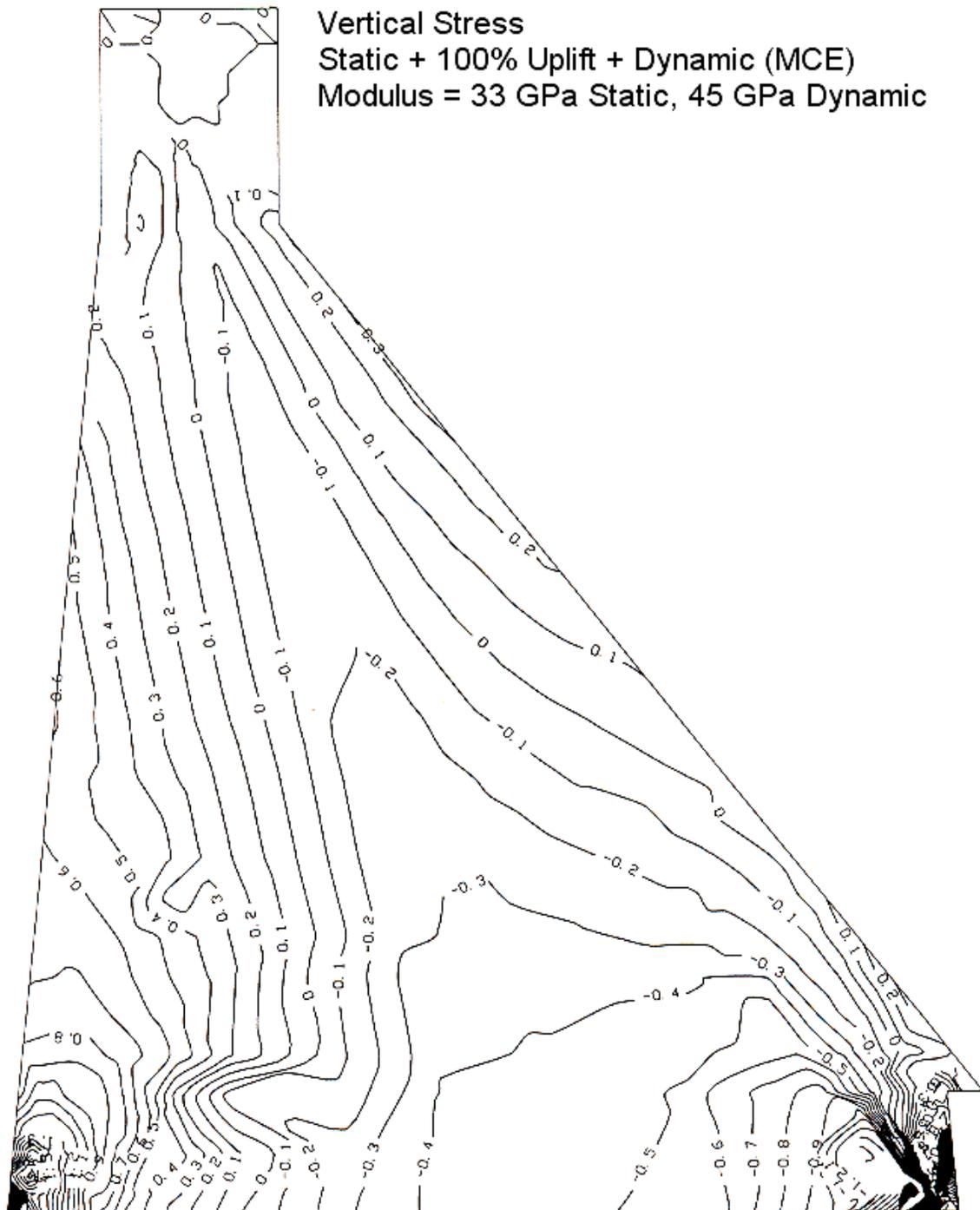
Figure 7
Vertical Stresses with Strain Softening – No Upstream Membrane

Vertical Stress (MPa)
Static +100% Uplift + Dynamic (MCE)
Modulus = 33 GPa Static, 45 GPa Dynamic
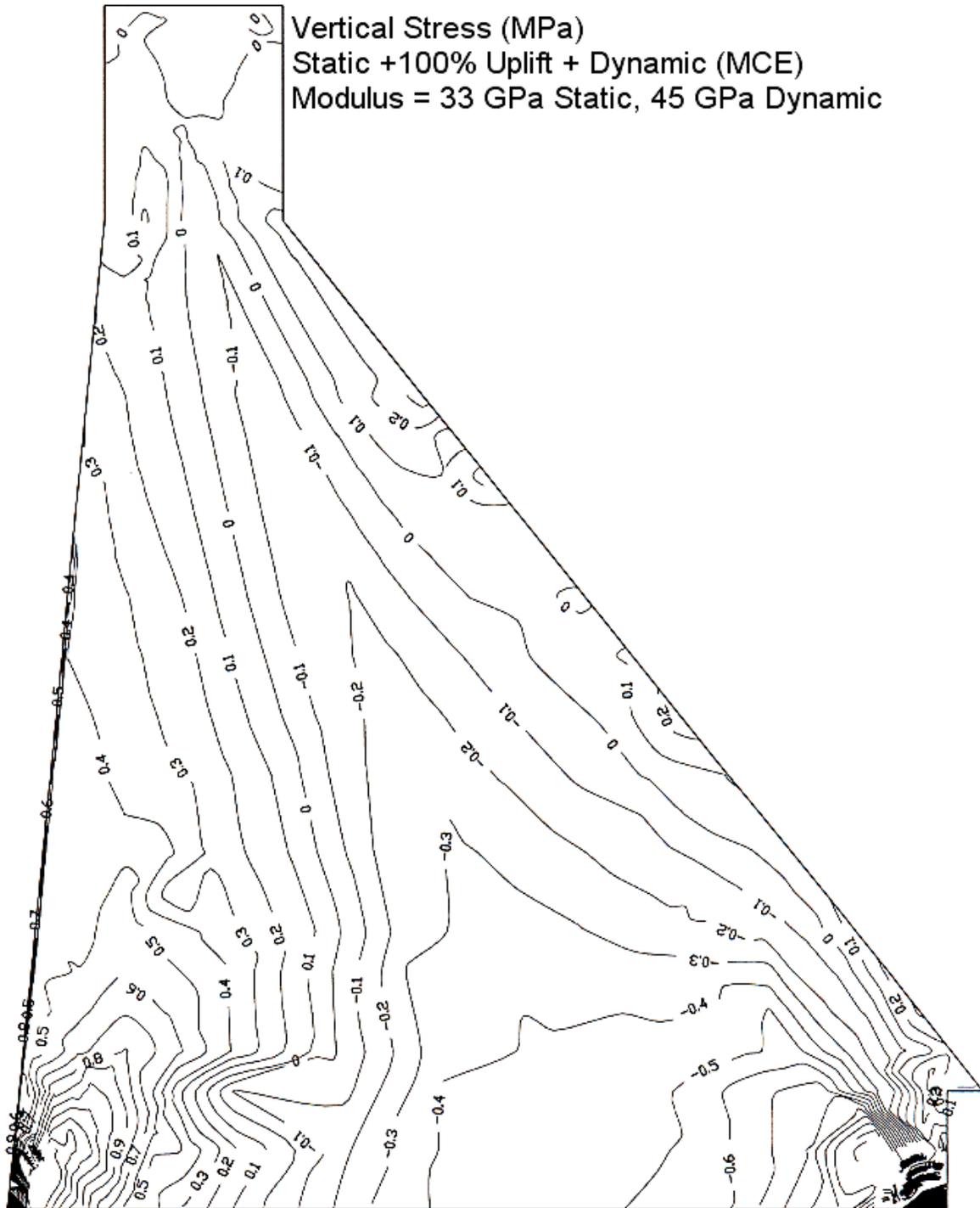
IMPROVED STABILITY & REDUCED STRESS DUE TO MEMBRANE

Upstream impervious membranes are a common and very successful way to simplify waterstops, assure total watertightness, and control seepage in both conventional and RCC dams. A significant benefit of the membrane system is its ability to eliminate uplift pressures, starting immediately at the upstream face. Drilled internal drains cannot effectively reduce uplift pressure directly at the face.

Experience has shown that seepage is virtually stopped with a proper membrane system. In the event that seepage did occur at a puncture or flaw in the membrane, the typical membrane system includes a drain immediately at the face, with provisions such as bedding along lift joints near the face to assure that any seepage will relieve itself into the drain rather than migrate into the dam. This system has proven to be extremely successful.

The stress analysis for Mujib considered the influence of the membrane system on stresses and stability. A separate rigid body analysis indicted that the membrane typically increased the factor of safety against sliding by about 10% to 20%, depending on the load condition, elevation in the dam, and membrane efficiency. The increase was greatest where the factor of safety was lowest.

When the membrane was considered in the FEM analysis a significant reduction of tensile stresses was realized in the heel. The reduction was approximately 1 MPa when the membrane was considered fully effective. The benefit was still significant at about 0.5 MPa when the membrane was conservatively considered to be only 50% effective. Figures 7, 8, and 9 show vertical stresses for the MCE load condition with strain softening and the membrane at 0% efficient (no membrane), 50% efficient, and 100% efficient.

Depending on the particular project and load conditions, a membrane can reduce dynamic tensile stresses from levels that would result in cracking to levels that avoid cracking. The benefit is greater for high dams. In addition, even if cracking were to occur, a membrane prevents uplift form entering separated lift joints, and it acts as a waterstop across the entire face of the dam, thereby preventing leaks.

For normal operating load conditions, the uplift reduction due to a proper membrane can change areas of undesirable tension at the upstream face and heel to areas of compression. This can result in the very desirable situation of compression for all parts of the dam under the normal operating condition, and often also for the flood condition.

The design for Mujib does not require a membrane above the gallery level. However, from a stress and stability perspective, the membrane will definitively improve stress and stability conditions if it is used for the entire dam, and it will assure total control of seepage for aesthetic and water conservation purposes.

Figure 8
**Vertical Stresses with Strain Softening – Upstream Membrane 50% Efficient**

Vertical Stress (MPa)
Static + Dynamic (MCE) +
50% Uplift Upstream
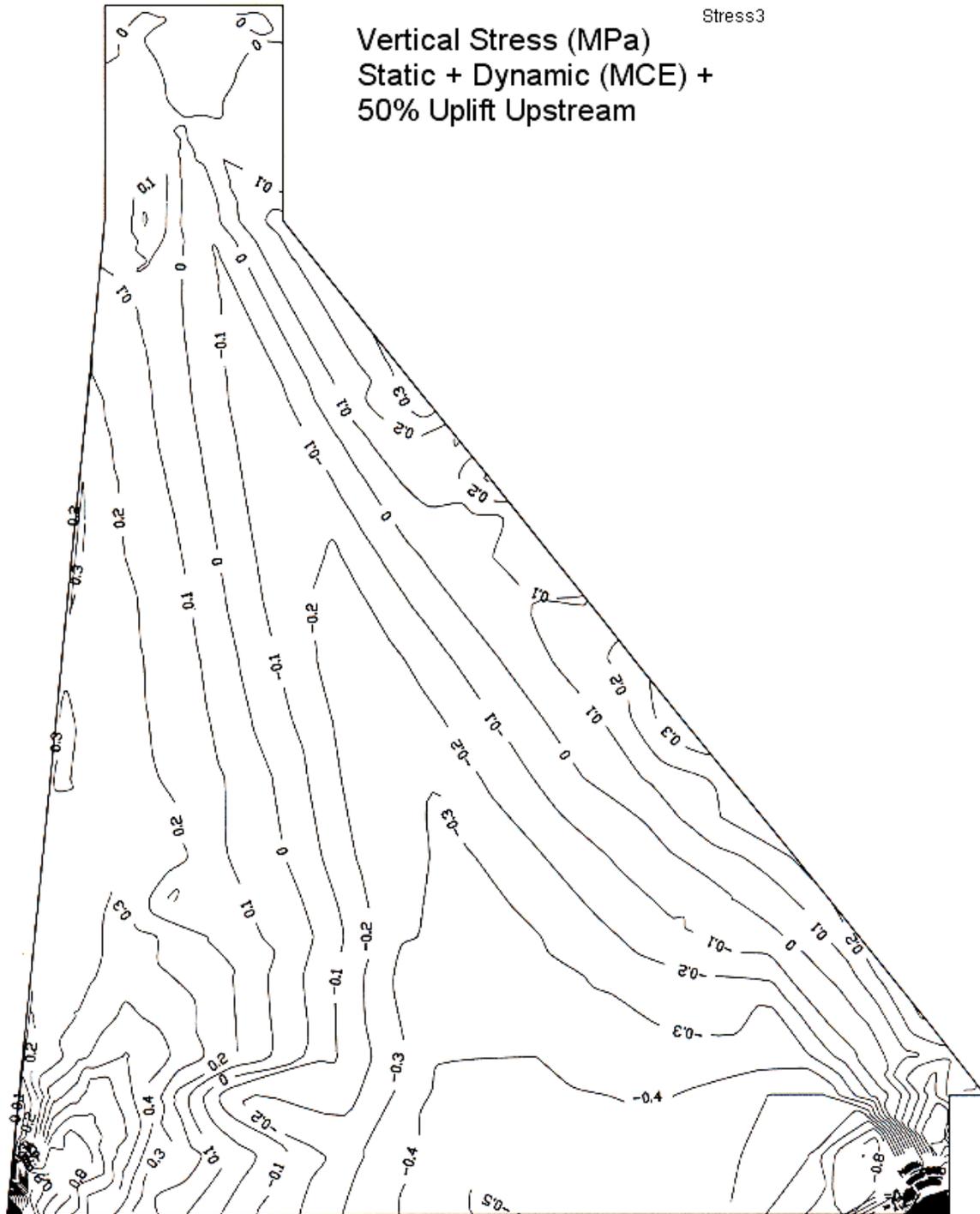
Stress3

Figure 9
Vertical Stresses with Strain Softening – Upstream Membrane 100% Efficient

Vertical Stress (MPa)
Static + Dynamic (MCE) + 0%
Uplift
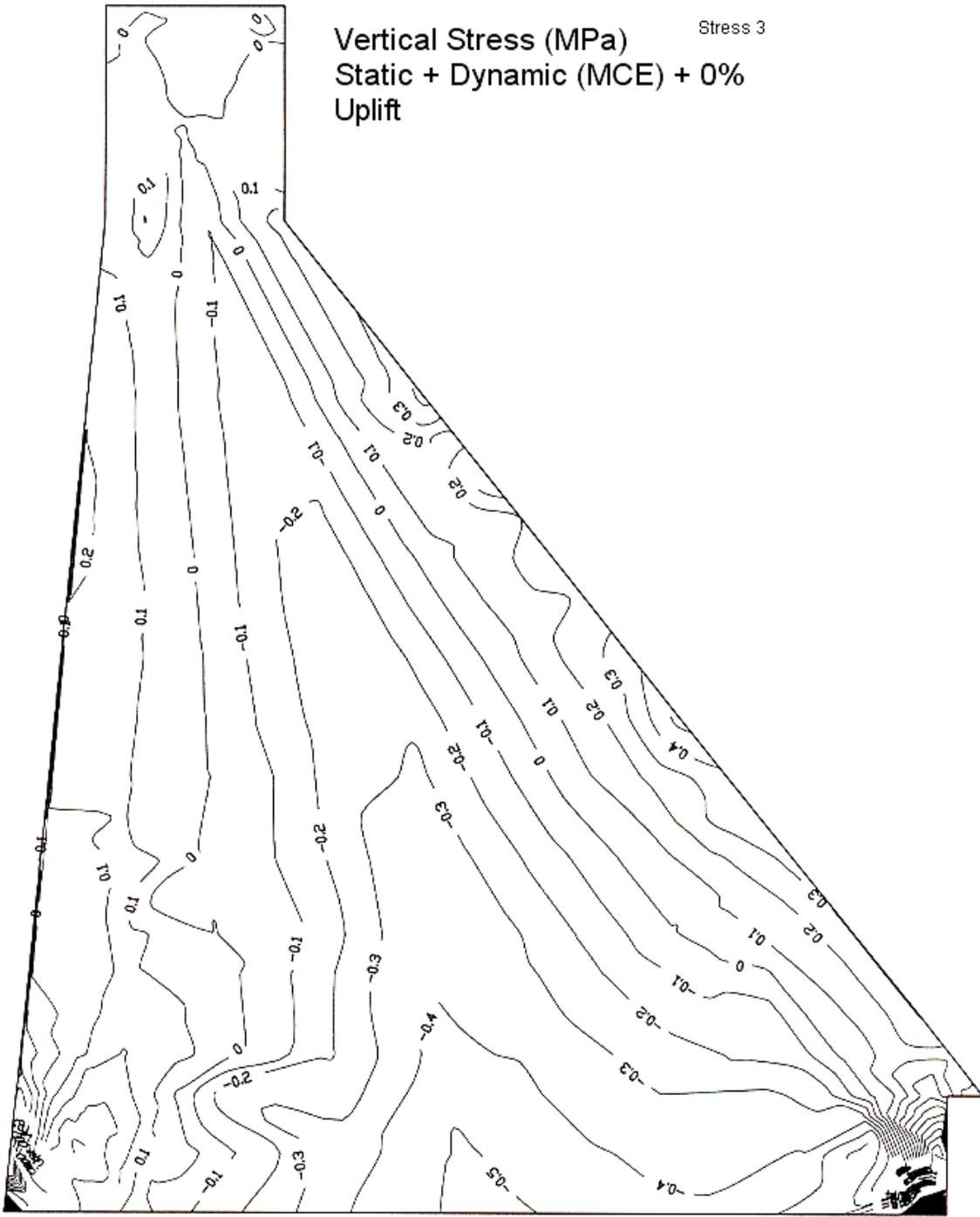
Stress 3

Figure 10
Vertical Stresses with Strain Softening – No Upstream Membrane – No Earthquake

Vertical Stress (MPa)
Static + 100% Uplift Upstream
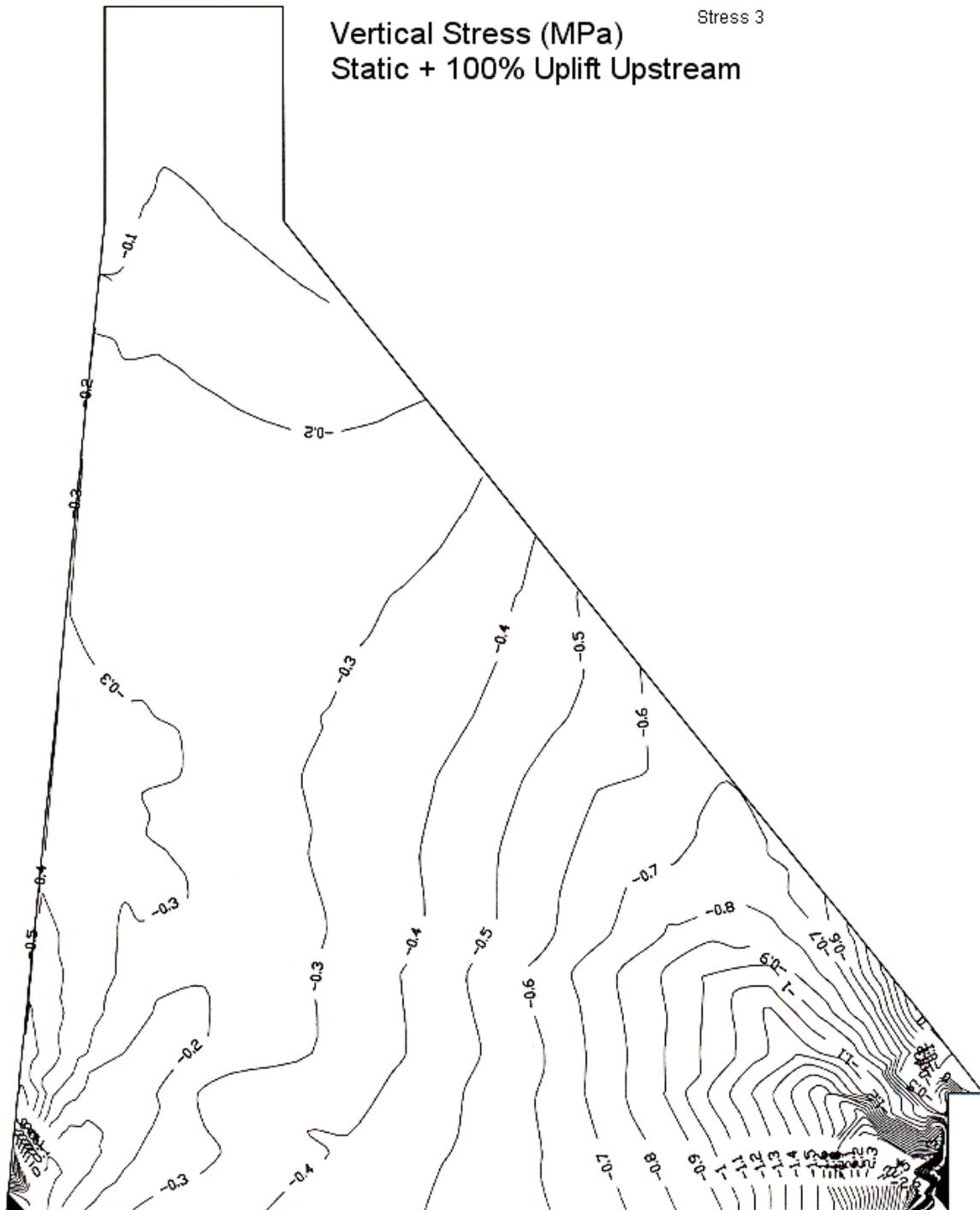
Stress 3

Figure 11
Vertical Stresses with Strain Softening – Upstream Membrane 50% Efficient

Vertical Stress (MPa)
Static + 50% Uplift Upstream
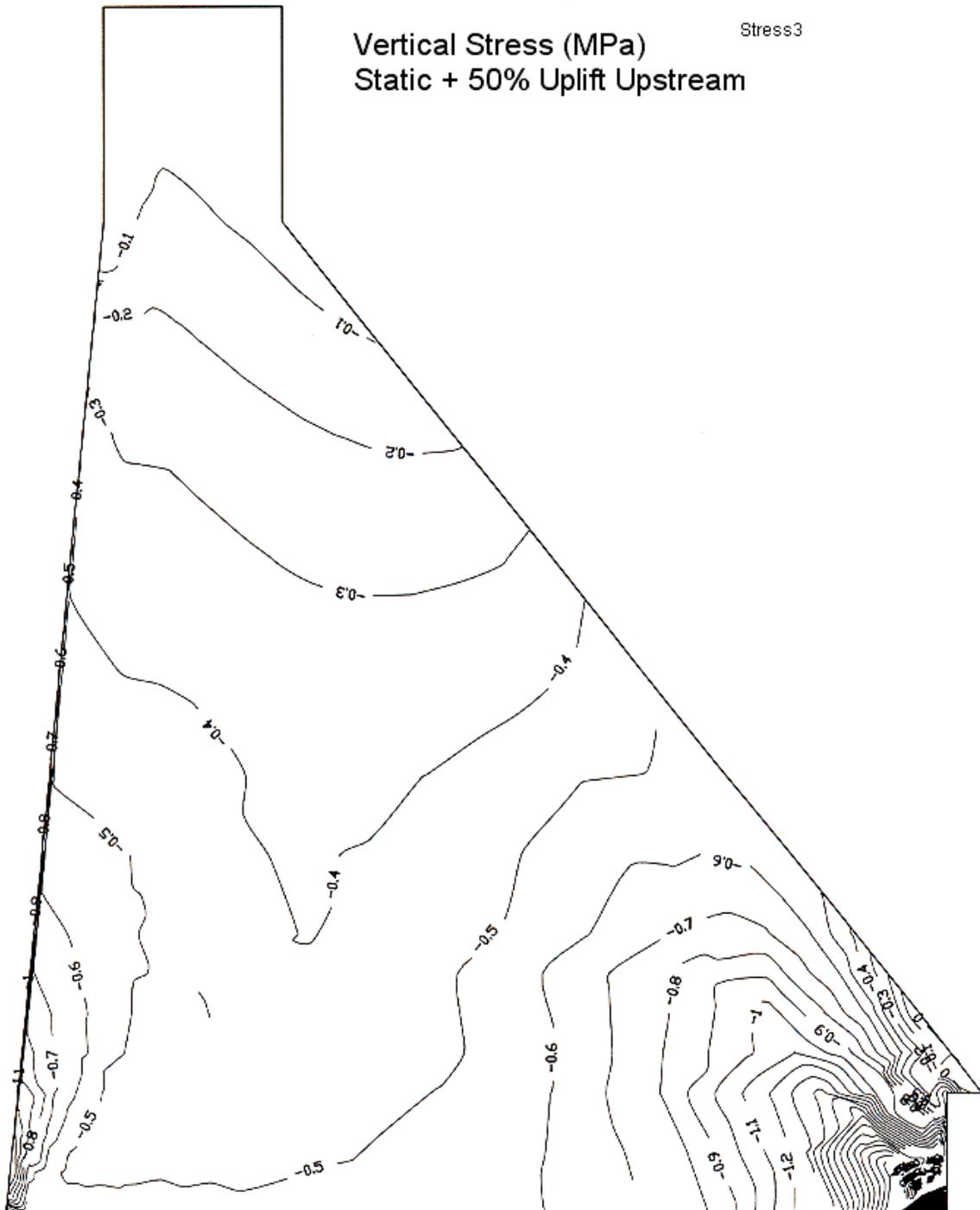
Stress3

CONCLUSIONS

Strain softening is a beneficial characteristic typical of low cementitious content RCC. When strain softening and the true non-linear stress-strain behavior of low cementitious content RCC is considered in state-of-the art stress analyses, the true stress condition is determined, with a significant reduction of peak stresses.

Computation methods and computer programs that do not define non-linear stress-strain behavior can still be used to easily accomplish the same end result by simply running through several iterations where the correct modulus value for stresses established at various elements in an FEM model are substituted for the previous values based on stress output of each iteration. No changes are required in the model, and usually only a few iterations are needed.

When the uplift reduction benefits of an upstream membrane and face drain system are considered, sliding stability increases and peak stresses are substantially reduced. The benefits are greatest in areas of highest stress.

Approximate Vertical Stresses
Near Upstream Face
(Same Region in the Dam)
Static + MCE

	Peak at Heel (MPa)	Upstream (MPa)	*Dynamic Tensile Strength (MPa)
No Strain Softening No Membrane	1.9	0.5 - 0.7	1.5 MPa
With Strain Softening No Membrane	1.4	0.3 – 0.5	1.5 MPa
With Strain Softening Membrane 50% Efficient	1.0	0.2 – 0.3	1.5 MPa
With Strain Softening Membrane 100% Efficient	0.7	0.1	1.5 MPa

* Dynamic Strength = 150% of Static Strength